Calculus Warm Up # 1- 5

Find the limits:

1. $\lim_{x \to 1} \dfrac{x^3 - 1}{x - 1}$

2. $\lim_{x \to 2} \dfrac{x^3 - 2x^2 + 2x - 4}{x - 2}$

3. $\lim_{x \to 7^+} \dfrac{x - 7}{|x - 7|}$

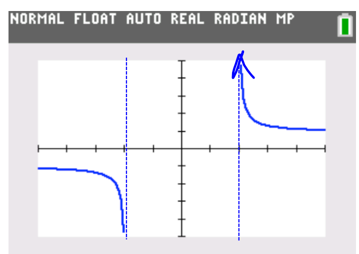
4. $\lim_{x \to 2^-} [\![3x - 1]\!]$

HW ?'s: p. 69

33. $\lim_{x \to 5^+} \dfrac{x - 5}{x^2 - 25}$

35. $\lim_{x \to 2^+} \dfrac{x}{\sqrt{x^2 - 4}}$

Can also check table



37. $\lim_{\Delta x \to 0^+} \dfrac{2(x + \Delta x) - 2x}{\Delta x}$

$\lim_{\Delta x \to 0^+} \dfrac{2x + 2\Delta x - 2x}{\Delta x}$

$\lim_{\Delta x \to 0^+} \dfrac{2\cancel{\Delta x}}{\cancel{\Delta x}}$

$= 2$

39. $\lim_{\Delta x \to 0^+} \dfrac{\frac{1}{x + \Delta x} - \frac{1}{x}}{\Delta x}$

41. $\lim_{x \to 0} \frac{|x|}{x}$

43. $\lim_{x \to 3} f(x), \quad f(x) = \begin{cases} \frac{x+2}{2}, & x \le 3 \\ \frac{12-2x}{3}, & x > 3 \end{cases}$

45. $\lim_{x \to 1} f(x), \quad f(x) = \begin{cases} x^3 + 1, & x < 1 \\ x + 1, & x \ge 1 \end{cases}$

$\lim_{x \to 1^-} (x^3 + 1) = 2$

$\lim_{x \to 1^+} (x + 1) = 2$

Since they match:

$\lim_{x \to 1} f(x) = 2$

47. $\lim_{x \to 3^-} 2[\![x - 3]\!]$

$= -2$

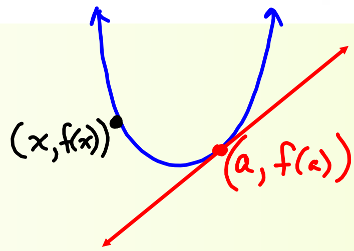
x	2.9	2.99
	$2[\![2.9-3]\!]$	$2[\![2.99-3]\!]$
	$2[\![-0.1]\!]$	$2[\![-0.01]\!]$
	$2(-1)$	$2(-1)$
	-2	-2

tangent line worksheet

1. $f(x) = x^2 + 3x + 4$

a) find the slope of the line tangent to $f(x)$ at any point $(a, f(a))$.

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$



$$\lim_{x \to a} \frac{x^2 + 3x + 4 - (a^2 + 3a + 4)}{x - a}$$

$$\lim_{x \to a} \frac{x^2 - a^2 + 3x - 3a}{x - a}$$

factoring top:

$(x + a)(x - a) + 3(x - a)$

$(x - a)(x + a + 3)$

$$\lim_{x \to a} \frac{\cancel{(x - a)}(x + a + 3)}{\cancel{x - a}}$$

$m = a + a + 3$

$m = 2a + 3$ at $(a, f(a))$

b) Find the slope of the tangent at $(0, 4)$.

$$m = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$$ or use $m = 2a + 3$ for $a = 0$

$\boxed{m = 3}$

c) Find the slope of the tangent where $x = -2$.

$$m = \lim_{x \to -2} \frac{f(x) - f(-2)}{x + 2}$$ or $m = 2(-2) + 3$

$\boxed{m = -1}$

d) Find the equation of the tangent line at $(-3,4)$.

$m = 2(-3)+3$

$= -3$

$\boxed{y - 4 = -3(x+3)}$

2. Find the equation of the line tangent to $f(x) = 3 - x^2$, at $(-2,-1)$

$m = \lim_{x \to -2} \frac{f(x) - f(-2)}{x+2} \ldots$

3. Find the equation of the tangent line to the graph of $y = \sqrt{x}$ at the point $(4,2)$.

4. Find the equation of the line tangent to $f(x) = \frac{1}{x}$, at the point $(2, \frac{1}{2})$.

5. The tangent line to the parabola $f(x) = x^2 + cx + d$ at the point $(1,2)$ has a slope of 3. Find c.

$$\lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = 3$$

and $f(1) = 2$

$1^2 + c(1) + d = 2$

$d = 1 - c$

sub in for d.

$$\lim_{x\to 1} \frac{x^2+cx+1-c-2}{x-1} = 3$$

$$\lim_{x\to 1} \frac{x^2-1+cx-c}{x-1} = 3$$

$$\lim_{x\to 1} \frac{(x-1)(x+1)+c(x-1)}{x-1} = 3$$

$$\lim_{x\to 1} \frac{\cancel{(x-1)}(x+1+c)}{\cancel{x-1}} = 3$$

$1+1+c = 3$

$\boxed{c = 1}$

2.1 - 2.2: Strategies to find limits

1. Look at a graph

★ Tues Quiz

2. Look at a table

3. Direct substitution if the function is continuous

4. Factor and cancel to create a new function

5. Rationalize the numerator or denominator

6. Find a common denominator to simplify compound fractions

2.1 - 2.2 part 2

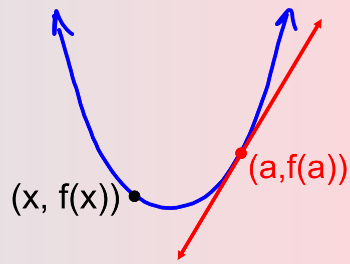
-Finding one-sided limits

-Determining limits that are not numeric values

-Finding the equation of a tangent line at a point on a curve

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Point - Slope Form
$y - f(a) = m(x - a)$



2.3 Continuity

-Determine continuity at a point

-Determine continuity on an open interval

-Determine continuity on a closed interval

-Investigate properties of continuity

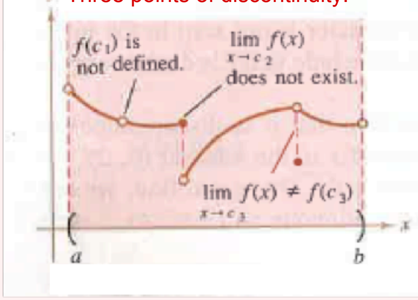
-Introduce the Intermediate Value Theorem

Definition of Continuity

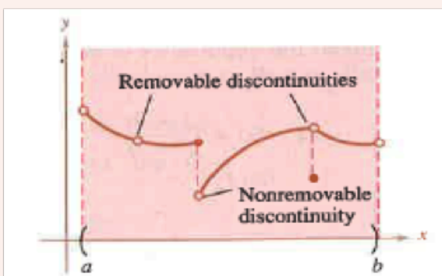
Continuity at a point: A function f is called continuous at c if the following 3 conditions are met

1. $f(c)$ is defined (it has a value, a point on the graph)
2. $\lim_{x \to c} f(x)$ *exists*
3. $\lim_{x \to c} f(x) = f(c)$

Three points of discontinuity:



A discontinuity is considered removable if by redefining that point it becomes continuous



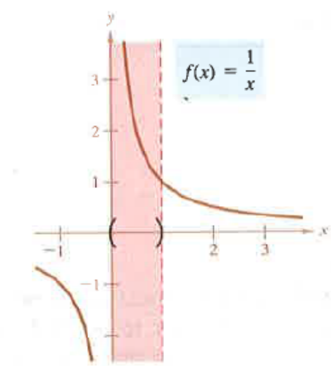
(a,b) means

"open interval from a to b"

Continuity on an Open Interval:
A function is called continuous on an open interval (a, b) if it is continuous at each point in the interval.

Decide whether the function is continous on the given interval:

(a) $f(x) = \dfrac{1}{x}, \quad (0, 1)$



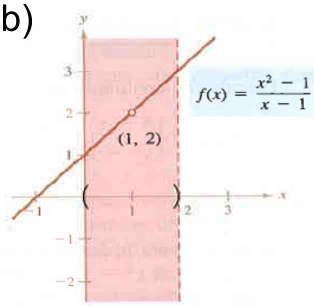
Continuous on (0, 1)

Continuity on an Open Interval:
A function is called continuous on an open interval (a, b) if it is continuous at each point in the interval.

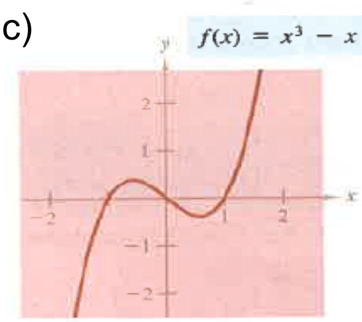
(b) $f(x) = \dfrac{x^2 - 1}{x - 1}, \quad (0, 2)$

(c) $f(x) = x^3 - x, \quad (-\infty, \infty)$

b)



c)

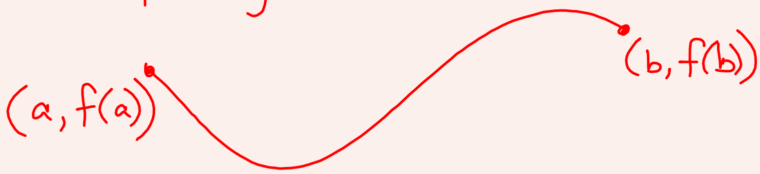


Continuity on a Closed Interval:
A function is continuous on a closed interval [a, b] if it is continuous on the open interval (a, b) and

$$\lim_{x\to a^+} f(x) = f(a) \quad \text{and} \quad \lim_{x\to b^-} f(x) = f(b).$$

approach a from right

approach b from the left

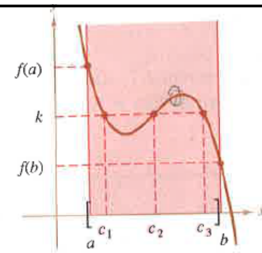


Properties of Continuity: If b is a real number and f and g are continuous at $x = c$, then the following functions are also continuous at c.

1. Scalar multiple bf
2. Sum and difference $f \pm g$
3. Product fg
4. Quotient f/g if $g(c) \neq 0$

Continuity of composite functions:
If g is continuous at c and f is continuous at $g(c)$, then the composite function given by $f(g(c))$ is continuous at c

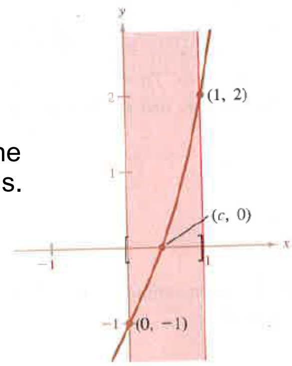
Intermediate Value Theorem: If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that $f(c) = k$



f is continuous.
(For k, there exist 3 c's.)

Use the Intermediate Value Theorem to show that *f(x)* has a zero on the interval [0, 1].

$f(x) = x^3 + 2x - 1$



f(x) is continuous on [0, 1] and since the outcomes for f(0) and f(1) change signs. we know f(x) crosses the x-axis.

HW: p. 75 (starts there)

1 - 45 eoo and 47